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**NONLINEAR DIFFERENCE EQUATIONS
AND POLYNOMIALS, ORTHOGONAL IN THE SOBOLEV
SENSE AND GENERATED BY CLASSICAL CHEBYSHEV
POLYNOMIALS OF DISCRETE VARIABLE**
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We consider the Cauchy problem for the nonlinear difference equation

$$\Delta y(x) = hf(x, y), \quad y(0) = y_0, \quad h > 0, \quad (1)$$

in which the function $f(x, y)$ we shall assume as given on the Cartesian product $\Omega_N \times \mathbb{R}$, where $\Omega_N = \{0, 1, \dots, N - 1\}$. Here and below $\Delta y(x) = y(x + 1) - y(x)$ is the finite difference operator.

It is required to approximate with a specified accuracy defined on Ω_{N+1} function $y = y(x)$, which represents the solution of problem (1).

Let us consider the system of polynomials $\tau_{1,n}^{\alpha,\beta}(x, N)$, ($n = 0, 1, \dots, N$), also defined on the net Ω_{N+1} and orthogonal in the Sobolev sense with respect to the following scalar product

$$\langle \tau_{1,n}^{\alpha,\beta}, \tau_{1,m}^{\alpha,\beta} \rangle = \tau_{1,n}^{\alpha,\beta}(0)\tau_{1,m}^{\alpha,\beta}(0) + \sum_{j=0}^{N-1} \Delta \tau_{1,n}^{\alpha,\beta}(j)\Delta \tau_{1,m}^{\alpha,\beta}(j)\mu(j),$$

where $\alpha, \beta > -1$, $\mu(x)$ – discrete weight function given by equality

$$\mu(x) = \mu(x; \alpha, \beta, N) = \frac{\Gamma(N)2^{\alpha+\beta+1}}{\Gamma(N + \alpha + \beta + 1)} \frac{\Gamma(x + \beta + 1)\Gamma(N - x + \alpha)}{\Gamma(x + 1)\Gamma(N - x)}.$$

Polynomials $\tau_{1,n}^{\alpha,\beta}(x, N)$ are determined through the classical Chebyshev polynomials of discrete variable by the following expressions:

$$\tau_{1,0}^{\alpha,\beta}(x, N) = 1, \quad \tau_{1,k+1}^{\alpha,\beta}(x, N) = \sum_{t=0}^{x-1} \tau_k^{\alpha,\beta}(t, N).$$

In current work we propose a new approach to the approximate solution of (1), based on decomposition of the function $y(x)$ on the grid Ω_{N+1} into a finite Fourier series by polynomials $\tau_{1,n}^{\alpha,\beta}(x, N)$:

$$y(x) = y(0) + \sum_{k=0}^{N-1} \hat{y}_{1,k+1} \tau_{1,k+1}^{\alpha,\beta}(x, N), \quad x \in \Omega_{N+1}.$$

where

$$\hat{y}_{1,k+1} = \sum_{t=0}^{N-1} \Delta y(t) \tau_k^{\alpha,\beta}(t, N) \mu(t), \quad (k \geq 0).$$

We note that the obtained results can be generalized to systems of difference equations of the form $\Delta y(x) = hf(x, y)$, $y(0) = y_0$, with $f = (f_1, \dots, f_m)$, $y = (y_1, \dots, y_m)$.

Moreover, problem (1) is of interest in connection with the fact, that the issue of the approximate solution of the Cauchy problem for ordinary differential equation

$$y'(x) = hf(x, y), \quad y(0) = y_0, \quad h > 0,$$

can be reduced to it.

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РАСХОДИМОСТЬ СУММ МОДУЛЕЙ БЛОКОВ РЯДОВ ФУРЬЕ – УОЛША ФУНКЦИЙ ОГРАНИЧЕННОЙ ВАРИАЦИИ

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Пусть f — функция ограниченной вариации на промежутке $[0, 1]$ и

$$\sum_{k=1}^{\infty} c_k w_k(x)$$

ее ряд Фурье–Уолша. Рассматривается система Уолша в нумерации Пэли. Как известно, ряды Фурье функций ограниченной вариации по тригонометрической системе всюду сходятся. В случае системы Уолша такие ряды сходятся в каждой точке непрерывности функции f (и даже в точках, где выполняется более слабое условие непрерывности), но могут расходиться в точках разрыва, хотя и ограничено. Пусть $\{n_j\}$ —