

In Memoriam Alexander Vasil'ev (1962–2016)

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On October 19, 2016, the mathematical community lost a valuable member, whose energy and passion for mathematics and his unique collegiality will be sorely missed by many of us around the globe. Alexander Vasil'ev (or Sasha, as many of us used to call him) was only 54 years old, and working hard as ever on one of his many projects. He is remembered as a kind and generous man, a devoted husband and father, an inspiring and supportive mentor, and a good friend to many who knew him (Fig. 1).

Sasha studied mathematics at Saratov State University, graduating with a Ph.D. in 1987. His thesis “Isoperimetric extremal problems in the theory of univalent functions” concerned different methods in estimating functionals in the classes of univalent functions, mainly, parametric and variational methods and optimal control approach [1,3].



Fig. 1 Sasha at his office in Bergen



Fig. 2 With Alexei Tochin and Alexander Bufetov (St. Petersburg, October 2016)

During the next few years, Sasha focused his research on the modulus method which has been applied to qualitative and quantitative extremal problems for conformal maps. Quadratic differentials play the main role in this approach. At the early stage he collaborated with a group of mathematicians from Saint Petersburg, see [4]. Later on he developed this method in many areas, see, e.g., [7,8], and summarized his results in the monograph [9].

It is very difficult to compress the many friendships and collaborations between so many people and Sasha into few sentences. We will always remember his smile when he first saw someone he already knew, his laughter, his hospitality, and his love of life. It was an honor and a privilege for all of us to be counted among his friends and collaborators. We are all fortunate to have known Sasha, sadly for not long enough (Fig. 2).

A brief overview of Alexander Vasil'ev's Vita (by Dmitri Prokhorov)

A natural extension of the modulus method led Sasha to extremal problems for quasiconformal maps and to the Teichmüller theory which has essential interactions with complex analysis, hyperbolic geometry, discrete groups and Lie groups, differential geometry, and many other fields. In particular, Vasil'ev and Hidalgo constructed a new compactification of the Teichmüller space of finitely generated Kleinian groups. They considered the modulus as a functional on the Teichmüller space and characterized the Teichmüller metric in terms of moduli, see [10,13].

In 1997, he defended his Habilitation thesis “Variational-geometric methods of solution of extremal and metric problems in the theory of conformal and quasiconformal mappings” at Novosibirsk State University.

Free boundary problems of fluid mechanics came into Sasha's research as an application of Loewner's theory to the Hele-Shaw flow, which describes the continual injection (or suction) of incompressible fluid into a free boundary domain. The first results in [5] on inherited geometry of the time-evolving fluid interface were published in 1998. One of the main achievements in the further articles [6, 11, 12] is that, in the difficult suction problem, there is a unique possible simply connected evolution. The monograph [14] contains an excellent exposition of the theory.

Between 2000 and 2005, Sasha was a professor at the Department of Mathematics at the Universidad Técnica Federico Santa María in Valparaíso, Chile. In 2006, he and his wife and collaborator Irina Markina moved to the University of Bergen, Norway. Together, they built a large research group in analysis and geometry, successfully graduating several Ph.D. students and managing a number of important European Union grants.

His earlier interest in Loewner's evolution transformed to deeper ideas at the crossroads between complex analysis and mathematical physics which in turn are found to be related to integrable systems. Sasha was one of four scientific leaders and program committee members of the Fall 2011 semester "Complex Analysis and Integrable Systems" at the Mittag-Leffler Institute, Djursholm, Sweden. With his coauthors in [15, 17, 21, 26, 26, 31], they found and studied relations between contour dynamics and Virasoro algebra. They pointed at solutions to the KP hierarchy which remain invariant on the Loewner–Kufarev trajectories embedded into the Sato–Segal–Wilson Grassmanian.

In recent years, Sasha had further expanded his research interests, in particular, to differential geometry and geometric control, sub-Riemannian geometry, SLE type stochastic processes in the plane, and several other directions. Sub-Riemannian manifolds often occur in constrained systems of mechanics. The Heisenberg group carries a natural sub-Riemannian structure. With his many collaborators, Sasha studied geodesics connecting two given points on odd-dimensional spheres respecting the Hopf fibration. They solved completely the geodesic boundary value problem for a 3-dimensional sphere also obtaining substantial partial results in the general case, and calculated the Carnot–Carathéodory distance. Importantly, they found the infinite-dimensional analogue of sub-Riemannian geometry on general locally compact manifolds and deduced several PDE of KdV type as the Arnold–Euler equations on the cotangent bundle, see, e.g., [18, 19, 22, 27, 30].

Expanding on the recent interest created by the Stochastic (Schramm–) Loewner Evolution (SLE), Sasha and his group proposed a model which describes deterministic and stochastic evolution of shapes in the complex plane and made progress in the geometry of the Loewner evolution for slits, see, e.g., [20, 23–25, 29]. The recent monograph [28] covers several of the topics mentioned.

Some recollections (by Björn Gustafsson)

I first met Alexander in the early years of the new millennium. He was a participant in a Mittag-Leffler programme (Probability and Conformal Mappings, 2001/2002) and I was a lecturer at KTH, Stockholm. Alexander suddenly showed up at my office and



Fig. 3 Sasha was known for being always at work on some project

wanted to discuss Hele-Shaw flow, a fluid dynamic free boundary problem which can be formulated in terms of conformal mappings, which was one of Alexander's areas of expertise. Driven by Alexander's energy, and including also contributions from Dmitri Prokhorov, the project that was initiated by our discussions resulted in a fine paper, published in the Proceedings of the American Mathematical Society with the three of us as authors [11]. Slightly later, Alexander and I wrote another paper on the subject, now for the same problem in higher dimensions [12] (Fig. 3).

Alexander was very generous, professionally and otherwise. He invited me to spend one month at his university in Chile, and I was able to bring my family as well. That sounded quite exiting, so we accepted, and came in the appointed period, starting by celebrating the Chilean Christmas 2003. The following month was a spectacular time, with many happy events such as tourist trips, wine parties, barbecues etc. On the professional side, the first thing that happened was that Alexander gave me a manuscript of a book, entitled "Conformal Analysis in Hele-Shaw Cells". He suggested that the book should be extended with me as a coauthor. After having studied the manuscript for a few days I accepted, and eventually this became the book "Conformal and Potential Analysis in Hele-Shaw Cells", published in 2006 by Birkhäuser [14].

After Sasha and Irina moved to Bergen, what followed was a very happy time for their family, and in addition a very productive period for them both. Alexander started (together with Irina) building research groups with doctoral students and applying for all sorts of European grants to support them. The ESF programme, HCAA, "Harmonic and Complex Analysis and Applications" was approved, and it was running in 2007–2012 with Alexander as the head of a steering committee of ten people. The programme worked very well, and was successful in all respects, including outstanding broad impact it had on the development of young mathematicians and their collaborations with more senior researchers. A large number of exciting conferences, workshops, and individual research visits were organized within the program. It was launched with



Fig. 4 Sasha with Björn Gustafsson and Vladimir Tkachev

a large meeting in Voss, a small village in the mountains near Bergen, and it ended with an equally well attended and exciting meeting in Tenerife, where, incidentally, Alexander's 50th birthday was memorably celebrated.

During the course of these years many things happened. For example, Alexander started the new journal "Analysis and Mathematical Physics" (first issue 2011). He wanted me to join him as a main editor, but I understood from previous experience how much work this would involve, so I declined. Therefore Alexander became the sole main editor, but I guess that Irina helped him a lot. Alexander and I coedited some other publications, for example, "Analysis and Mathematical Physics", Birkhäuser, 2009 [33], but, as usual I would say, it ended up with Alexander doing almost all the work. Another major event was the Mittag-Leffler programme "Complex Analysis and Integrable Systems", organized, as part of the ESF programme, in the fall of 2011. There were four leaders of the programme (besides Alexander, Håkan Hedenmalm, Nikolai Makarov and myself), but of course, Alexander was the true leader, taking care of all the details and particulars.

The last time I met Alexander was at the 27th Nordic Congress of Mathematicians, held at Stockholm University, on 16–20 March, 2016. This was at the same time the one hundred years celebration of the Mittag-Leffler institute. The photo (Fig. 4) is from this occasion.

I am glad that I got the above described fifteen years with Alexander, it has been one of the most joyful and productive periods in my life. And I am equally sad that it is now over, much too early. It is difficult to accept that I will not meet Alexander anymore, but his happy and optimistic face will forever stay strong in my mind.

Remembering Sasha Vasil'ev (by Armen Sergeev)

For me it is hard to imagine any other man so full of life as Sasha. His optimism manifested itself in all what he did—in mathematics, meeting friends and colleagues, in his family. It was clearly felt by everybody who knew him.

I got acquainted with Sasha and Irina at the Bialowiezha conference on Geometric Methods in Physics in 2011. From the first days of our friendship I got an impression that we knew each other for many years.

Our next meeting happened during the European Mathematical Congress in Krakow in 2012. He proposed to write a joint review on the Teichmüller theory for “Russian Mathematical Surveys”. This work [32] we dedicated to the memory of Andrei Alexandrovich Gonchar—a man whom we loved and respected a lot and whom I regard as one of my teachers.

The paper consisted of two parts. The first one, devoted to classical Teichmüller spaces, was written by Sasha and the second one (on the universal Teichmüller space) by me. The paper started from a historical introduction, written by Sasha alone, which I read with great interest. It was devoted to three mathematicians who played a crucial role in forming a new mathematical discipline called now Teichmüller theory, namely Oswald Teichmüller, Lars Ahlfors and Lipman Bers. The life trajectories of those people were completely different and it probably explains why it is so interesting to follow them. Sasha managed (in short biographies) to single out the main features of their characters and bring them to the reader, incorporating the dry facts into the “live story” of people who lived in those complicated times.

Sasha invited me several times to visit him in Bergen and at last I came there in the spring of 2015. I was especially impressed by the first day of my stay when we went together sightseeing around Bergen. It was an unforgettable trip, a great show of rocky mountains, deep fiords and full-flowing waterfalls. At the end of the day we were sitting in Sasha's house drinking beer skillfully prepared by Sasha himself (in making beer he was as masterful and professional as in all of his other endeavors).

The same year Sasha and Irina came to Moscow and it was another highlight of this wonderful year. Together with Andrei Bogatyrev we went to the birth place of Pafnutii Lvovich Tchebyshev whom we consider as one of the founders of Russian mathematical school. Tchebyshev estate in Kaluga region (approximately 120 km south of Moscow) was unfortunately not preserved, however his grave in the Tchebyshev family church survived. In the underground vault of this church in an old village (Spas-Prognan) visitors can see the graves of Tchebyshev and two of his brothers, generals in the Russian army.

Our last meeting occurred in the summer of 2016 during a conference in Kazan not long before Sasha passed away. As always, Sasha was full of joyful plans for the future and it was difficult to imagine that he had any health problems. We were discussing the plans for our future cooperation and thought that we had many interesting things to do in the coming years. Unfortunately, these plans cannot be realized without Sasha. An important part of my life has vanished with Sasha leaving only the memory of our joint work and wonderful moments of our meetings.

Björn Gustafsson, Dima Khavinson, Dmitri Prokhorov, Armen Sergeev and Razvan Teodorescu

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