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РАВНОСХОДИМОСТЬ ОДНОГО ИНТЕГРАЛЬНОГО ОПЕРАТОРА

EQUICONVERGENCE OF AN INTEGRAL OPERATOR

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This report is devoted to a research equiconvergence of expansions into trigonometrical Fourier series with respect to their associated eigenfunctions of one class of integral operators whose kernels take various constant values in and out of the square inscribed in a single square. The research of equiconvergence of spectral expansions represents the developing direction which foundation was laid in the articles of V. A. Steklov, E. Gobson, A. Haar . But, the Theorem of equiconvergence for the integral operator was for the first time received by A. P. Khromov.

He considered a case when some derivative kernels have a rupture of the 1st kind on the line t = x. Then he researched a new class of integral operators when this property of kernels is watched on lines t = x and t = 1 − x. This class is remarkable by the fact that it makes possible to study the behavior of a resolvent of Fredholm in case of great values of spectral parameter that is important in case of spectrum analysis of such operators.

We will consider the integral operator the kernel of which accepts constant values inside and outside the square inscribed in a single square (he generalizes the operator given in [1-2]). Its kernel is provided in a figure 1.

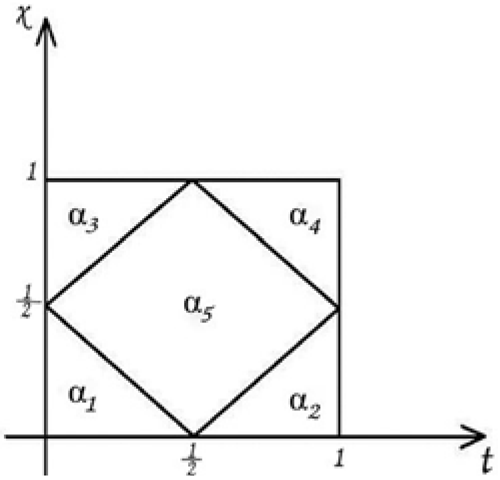

Figure 1

This integral operator in a scalar look can be taken into the integral operator in a vector look. This property is defined in a lemma 1.

Lemma 1.

$$If\ \ y(x) = Af, then\ z(x) = \int_0^{1/2} B(x,t)g(t), 0 \le x \le 1/2, where$$

$$z(x) = \begin{pmatrix} z_1(x) \\ z_2(x) \end{pmatrix} = \begin{pmatrix} y(x) \\ y(1/2+x) \end{pmatrix};\ \mathrm{g(x)} = \begin{pmatrix} \mathrm{g}_1(\mathrm{x}) \\ \mathrm{g}_2(\mathrm{x}) \end{pmatrix} = \begin{pmatrix} \mathrm{f(x)} \\ \mathrm{f(1/2+x)} \end{pmatrix}$$

$$B(x,t) = \begin{pmatrix} B_{11}(x,t) & B_{12}(x,t) \\ B_{21}(x,t) & B_{22}(x,t) \end{pmatrix} = \begin{pmatrix} A(x,t) & A(x,1/2+t) \\ A(1/2+x,t) & A(1/2+x,1/2+t) \end{pmatrix}.$$

Lemma 2.

$$If\ \ y(x) = Af, then$$

$$\begin{pmatrix} y(x) \\ y(1/2+x) \end{pmatrix} = \begin{pmatrix} 0 & \alpha_5 \\ \alpha_3 & 0 \end{pmatrix} \int_0^x \begin{pmatrix} f(t) \\ f(1/2+t) \end{pmatrix} dt + \begin{pmatrix} 0 & \alpha_2 \\ \alpha_5 & 0 \end{pmatrix} \int_x^{1/2} \begin{pmatrix} f(t) \\ f(1/2+t) \end{pmatrix} dt +$$

$$+ \begin{pmatrix} \alpha_1 & 0 \\ 0 & \alpha_5 \end{pmatrix} \int_0^{1/2-x} \begin{pmatrix} f(t) \\ f(1/2+t) \end{pmatrix} dt + \begin{pmatrix} \alpha_5 & 0 \\ 0 & \alpha_4 \end{pmatrix} \int_{1/2-x}^{1/2} \begin{pmatrix} f(t) \\ f(1/2+t) \end{pmatrix} dt, x \in [0,1/2].$$

We will enter a resolvent of Fredholm of this operator.

$$R_\lambda(A)f = (E - \lambda A)^{-1} Af - \text{resolvent of Fredholm}$$

Theorem 1.

$$\text{If } y = R_\lambda(A)f, v(x) = \left(y(x), y(\frac{1}{2}+x), y(\frac{1}{2}-x), y(1-x) \right) - \text{solution of the system (1)-(2)}$$

$$\begin{cases} v'(x) = \lambda D v(x) + D m(x) & (1) \\ P_0 v(0) + P_1 v(1/2) = 0 & (2) \end{cases}$$

where

$$\begin{aligned} & v(x) = (v_1(x), v_2(x))^T \\ & v_1(x) = z\ (x) \\ & v_2(x) = z(1/2 - x) \\ & m(x) = (g(x), g\ (1/2 - x))^T \end{aligned}, \quad D = \begin{pmatrix} 0 & a & b & 0 \\ c & 0 & 0 & d \\ -b & 0 & 0 & -a \\ 0 & -d & -c & 0 \end{pmatrix}, \begin{cases} a = \alpha_2 - \alpha_2 \\ b = \alpha_5 - \alpha_1 \\ c = \alpha_3 - \alpha_5 \\ d = \alpha_4 - \alpha_5 \end{cases}.$$

And sufficient condition, theorem 2.

Theorem 2.

$$\text{If } v(x) = (v_{11}(x), v_{12}(x), v_{21}(x), v_{22}(x))^T, \text{satisfy (1)-(2)}$$

and the corresponding homogeneous equation has only the trival solution

$$R_\lambda(A)f = \begin{cases} v_{11}(X), 0 \le X \le 1/2 \\ v_{12}(X - 1/2), 1/2 \le X \le 1 \end{cases}$$

Also we will prove a lemma 3.

Lemma 3.

$$\text{If } a \ne 0, d \ne 0, (d+b)^2 - 4ac \ne 0 \text{ the matrix } D \text{ is similar}$$

$$\text{for diagonal matrix } D_1 = diag(\omega_1, \omega_2, -\omega_2, -\omega_1), \text{where } \omega_1 \ne \omega_2.$$

Then we find a nondegenerate matrix Γ such that on the main diagonal there are eigenvalues.We'll make some more replacements

And we'll get the system (3)-(4).

$$\text{Let } v = \Gamma h$$

$$\begin{cases} h'(x) = \lambda D_1 h(x) + \Gamma^{-1} D m(x) = \lambda D_1 h(x) + m_1(x) & (3) \\ U(h) = P_0 \Gamma h(0) + P_1 \Gamma h(1/2) = 0 & (4) \end{cases}$$

The solution of the system (3)-(4) helps to prove the main theorem:

Theorem 3. (Theorem about equiconvergence)

$$\forall f(x) \in L[0,1] \quad \varepsilon \in (0,1/4)$$

$$\lim_{r\to\infty} \left\| S_r(f,x) - \sum_{j=1}^{\mu} \gamma_{kj} \frac{1}{\omega_j} \sigma_{r|\omega_j|}(m_1, \frac{k-1}{2}) \right\|_{[\frac{k-1}{2}-\varepsilon, \frac{k}{2}-\varepsilon]} = 0$$

$$where, S_r(f,x) = -\frac{1}{2\pi i} \int_{|\lambda|=r} R_\lambda(A) f \, d\lambda, \quad k = 1,2.$$

$S_r(f,x)$ - partial sum of Fourier serieson charactestical assosiated functions $|\lambda_k| < r$ of operator A.

$\sigma_r(f,x)$ - partial sum of Fourier series on assosiated functions of operator $u'(x)$.

m_{jk} - components of column $m_1(x)$.

$u(0) = u(1/2)$ - boundary value problem for own values $\lambda_k^0, |\lambda_k^0| < r$

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