and sufficient that the function $u(z)$ be analytic on the disk ($|z| < 1$) and that the following inequalities be valid:

$$0 < \inf |u(z)|, \quad \sup |u(z)| < \infty.$$

Theorem 1. *Let $\{w_n\}_{n\geq 0}$ be the Walsh system, $\{r_k\}_{k\geq 0}$ be the Rademacher system and*

$$f = \sum_{k=0}^{\infty} a_k r_k, \qquad \sum_{k=0}^{\infty} |a_k|^2 < \infty.$$

If the analytic function

$$\hat{f}(z) = \sum_{k=0}^{\infty} a_k z^k, \qquad |z| < 1,$$

belongs to $G(H^\infty)$, then the affine system of Walsh type $\{f_n\}_{n\geq 0}$ is Riesz bases in $L^2(0,1)$.

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A CLASS OF ANALYTICAL UNIVALENT FUNCTIONS DEFINED BY AN INTEGRAL OPERATOR[1]

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Let RV be the class of analytical functions that are univalent in the unit circle $U = \{z \in C \colon |z| < 1\}$ and such that their power series expansions have the form

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n. \tag{1}$$

Denote by RVT the subclass of functions $f \in RV$ with nonnegative coefficients in expansion (1):

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n, \quad a_n \geq 0, \quad n \geq 2. \tag{2}$$

For $\alpha > -1$, on the set of functions $f \in RV$, define the interal operator

$$F^\alpha(z) = (F^\alpha f)(z) = \frac{\alpha+1}{z^\alpha} \int_0^z \delta^{\alpha-1} f(\delta)\, d\delta, \quad z \in U.$$

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It is not hard to verify the following formula:

$$(F^\alpha f)(z) = z - \sum_{n=2}^{\infty} \left(\frac{\alpha+1}{\alpha+n}\right) a_n z^n, \quad z \in U.$$

For

$$\alpha > -1, \quad \lambda \geq 0, \quad \beta \geq 0, \quad \theta \geq 0, \tag{3}$$

denote by $RVT(\alpha, \beta, \lambda, \theta)$ the set of functions $f \in RVT$ satisfying the condition

$$\left| \frac{\beta\left[z^2(F^\alpha(z))'' - \theta z(F^\alpha(z))' - z\right]}{1 + \lambda\left[z^2(F^\alpha(z))'' - \theta z(F^\alpha(z))' - z\right]} \right| < 1, \quad z \in U.$$

Some other similar classes of functions from RVT were studied by Abudul Hussein and Buti [1,2], Atshan and Buti [3,4], Goel and Sahi [5], Slivarman [6], Pashkouleva and Vasilev [7], Kulkarni, Aouf, and Joshi [8], Juneja and Mogra [9].

The main results of this talk are contained in the following statements, where we suppose that conditions (3) hold.

Theorem 1. *A function $f \in RVT$ belongs to the class $RVT(\alpha, \beta, \lambda, \theta)$ if and only if*

$$\sum_{n=2}^{\infty} \left(\frac{\alpha+1}{\alpha+n}\right) n(n-1-\theta)(\beta+\lambda) a_n \leq 1.$$

As a consequence of this theorem, we can conclude that coefficients of expansion (2) of a function $f \in RVT(\alpha, \beta, \lambda, \theta)$ satisfy the following estimates:

$$a_n \leq \frac{1}{\left(\frac{\alpha+1}{\alpha+n}\right)} n(n-1-\theta)(\beta+\lambda), \quad n \geq 2.$$

Theorem 2. *For a function $f \in RVT(\alpha, \beta, \lambda, \theta)$, the following estimates are valid*:

$$r - \frac{1}{2\left(\frac{\alpha+1}{\alpha+2}\right)(1-\theta)(\beta+\lambda)} r^2 \leq |f(z)| \leq$$

$$\leq r + \frac{1}{2\left(\frac{\alpha+1}{\alpha+2}\right)(1-\theta)(\beta+\lambda)} r^2, \quad 0 < |z| = r < 1, \tag{4}$$

For the function

$$f(z) = z - \frac{1}{2\left(\dfrac{\alpha+1}{\alpha+2}\right)(1-\theta)(\beta+\lambda)} z^2,$$

both inequalities in (4) *turn into equalities.*

Theorem 3. *Let* $f_1(z) = z$ *and*

$$f_n(z) = z - \frac{1}{\left(\dfrac{\alpha+1}{\alpha+n}\right) n(n-1-\theta)(\beta+\lambda)} z^n, \quad n \geq 2.$$

A function f *belongs to the class* $RVT(\alpha, \beta, \lambda, \theta)$ *if and only if this function can be represented in the form*

$$f(z) = \sum_{n=1}^{\infty} \sigma_n f_n(z), \quad |z| < 1,$$

where $\sigma_n \geq 0$, $n \geq 1$; *moreover,* $\sum\limits_{n=1}^{\infty} \sigma_n = 1$.

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