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ON CONSTRUCTION OF RIESZ BASES USING WALSH TYPE AFFINE SYSTEMS IN THE SPACE $L^2(0,1)$[1]

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Suppose that the periodic function $f \in L^2(0,1)$ satysfied the condition $\int\limits_0^1 f(t)\,dt = 0$. For any $n \in \mathbb{N}$ using the binary representation $n = \sum\limits_{\nu=0}^{k-1} \alpha_\nu 2^\nu + 2^k$ we set

$$f_n(t) = W^\alpha f(t) = W_{\alpha_0} \dots W_{\alpha_{k-1}} f(t). \tag{1}$$

Besides, we set $f_0(t) \equiv 1$. The system $\{f_n\}_{n\geq 0} = \{W^\alpha f\}_{\alpha \in \mathbb{A}}$ ($\mathbb{A} = \bigcup_{k=0}^{\infty}\{0,1\}^k$) is the affine system of Walsh type, where $\alpha \in \mathbb{A}$, $\alpha = (\alpha_0, \dots, \alpha_{k-1})$ and

$$W_0 f(t) = f(2t), \qquad W_1 f(t) = r(t) f(2t).$$

It is easy to show that the Walsh – Paley system $\{w_n\}_{n\geq 0}$ is affine system.

We consider the problem: under which conditions on the function f an affine system of Walsh type $\{f_n\}_{n\geq 0}$ is a Riesz basis in the space $L^2(0,1)$, i.e. $f_n = Ae_n$, $n \geq 0$, where A is an invertible and a bounded (together with A^{-1}) operator in $L^2(0,1)$ and $\{e_n\}_{n\geq 0}$ is an orthonormal basis.

In our work we indicate a method for constructing Riesz bases. This method is shown as follow: suppose that H^∞ is the Banach algebra of analytic functions on the open unit disk, $G(H^\infty)$ is the group of invertible elements of the algebra H^∞. Note that for u to be belong to $G(H^\infty)$, it is necessary

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and sufficient that the function $u(z)$ be analytic on the disk ($|z| < 1$) and that the following inequalities be valid:

$$0 < \inf |u(z)|, \quad \sup |u(z)| < \infty.$$

Theorem 1. *Let $\{w_n\}_{n\geq 0}$ be the Walsh system, $\{r_k\}_{k\geq 0}$ be the Rademacher system and*

$$f = \sum_{k=0}^{\infty} a_k r_k, \qquad \sum_{k=0}^{\infty} |a_k|^2 < \infty.$$

If the analytic function

$$\hat{f}(z) = \sum_{k=0}^{\infty} a_k z^k, \qquad |z| < 1,$$

belongs to $G(H^\infty)$, then the affine system of Walsh type $\{f_n\}_{n\geq 0}$ is Riesz bases in $L^2(0,1)$.

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A CLASS OF ANALYTICAL UNIVALENT FUNCTIONS DEFINED BY AN INTEGRAL OPERATOR[1]

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Let RV be the class of analytical functions that are univalent in the unit circle $U = \{z \in C\colon |z| < 1\}$ and such that their power series expansions have the form

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n. \tag{1}$$

Denote by RVT the subclass of functions $f \in RV$ with nonnegative coefficients in expansion (1):

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n, \quad a_n \geq 0, \quad n \geq 2. \tag{2}$$

For $\alpha > -1$, on the set of functions $f \in RV$, define the interal operator

$$F^\alpha(z) = (F^\alpha f)(z) = \frac{\alpha+1}{z^\alpha} \int_0^z \delta^{\alpha-1} f(\delta)\, d\delta, \quad z \in U.$$

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