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GEOMETRIC PROPERTIES OF WRIGHT FUNCTION

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The Wright function is define by

$$W_{\lambda,\mu}(z) = \sum_{n=0}^{\infty} \frac{z^n}{n!\,\Gamma(\lambda\, n + \mu)}, \quad \lambda > -1,\ \mu \in \mathbb{C}. \tag{1}$$

This function was studied in connection with the partitions of the natural numbers [1]. For $\lambda > -1$, the Wright function is an entire function of z and has been used widely in the asymptotic theory of partitions, Mikusinski operational calculus, integral transforms and in fractional differential equations [2]. For $\lambda = 1$, $\mu = \nu + 1\,(\nu > -1)$, the function $W_{\lambda,\mu}$ can be represented in terms of the Bessel functions. Also, Wright function generalizes various functions like Array function, Whittakar function, (Wright-type) entire auxiliary functions etc.

Let $\mathcal{A}$ denote the class of all analytic functions in the open unit disk $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ having the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n\, z^n, \quad z \in \mathbb{D}. \tag{2}$$

We denote by $\mathcal{S}$, the subclass of $\mathcal{A}$ which are also univalent in $\mathbb{D}$. A function $f \in \mathcal{A}$ is called starlike, denoted by $\mathcal{S}^*$ if $tw \in f(\mathbb{D})$ whenever $w \in f(\mathbb{D})$ and $t \in [0,1]$. A function $f \in \mathcal{A}$ is called starlike function, if $\mathrm{Re}\,(zf'(z)/f(z))\,hm > 0$, $z \in \mathbb{D}$. Further, we denote by $\widetilde{\mathcal{S}}^*(\alpha)$, $0 < \alpha \leq 1$, the class of strongly starlike functions of order α, which is defined by

$$\widetilde{\mathcal{S}}^*(\alpha) = \left\{ f \in \mathcal{A} : \left| \arg\left(\frac{zf'(z)}{f(z)} \right) \right| < \frac{\alpha\pi}{2}, \quad z \in \mathbb{D} \right\}. \tag{3}$$

Note that $\widetilde{\mathcal{S}}^*(1) \equiv \mathcal{S}^*$. An analytic function f is called close-to-convex in $\mathbb{D}$, if complement of $f(\mathbb{D})$ can be written as the union of non-intersecting half-lines. An analytic function f is said to be close-to-convex with respect to a fixed starlike function g, denoted by $\mathcal{C}_g$, if Re $(zf'(z)/g(z)) > 0$, $z \in \mathbb{D}$.

In this paper, we consider the following normalized form of Wright function

$$\mathbb{W}_{\lambda,\mu}(z) = z\Gamma(\mu)W_{\lambda,\mu}(z) := \sum_{n=0}^{\infty} \frac{\Gamma(\mu)z^{n+1}}{n!\,\Gamma(\lambda n+\mu)}, \quad (\lambda > -1,\ \mu > 0,\ z \in \mathbb{D}). \tag{4}$$

We note that, the normalized Wright function $\mathbb{W}_{\lambda,\mu}$ belong to the family $\mathcal{A}$. The function $\mathbb{W}_{\lambda,\mu}$ studied recently by author [3].

The special functions plays an important role in function theory, especially the hypergeometric function in the solution by L. De-Branges [4] of the famous Bieberbach conjecture. Several researchers studied classes of analytic functions involving special functions $\mathcal{F} \subset \mathcal{A}$, to find different conditions such that the members of $\mathcal{F}$ have certain geometric properties such as univalency, starlikeness or convexity in $\mathbb{D}$. In this presentation, we discuss the following geometric properties and inequalities of the Wright function $\mathbb{W}_{\lambda,\mu}$.

Theorem 1. *If $\lambda \geq 1$, $\mu \geq 1$ and $\Gamma(\lambda+\mu) \geq 2\Gamma(\mu)$, then $\mathbb{W}_{\lambda,\mu}$ is close-to-convex with respect to $g(z) = z/(1-z)$.*

Theorem 2. *For each $\lambda \geq 1$, $\mu \geq 1$ such that $\Gamma(\lambda+\mu) \geq 4\Gamma(\mu)$, the function $\mathbb{W}_{\lambda,\mu}$ is starlike in $\mathbb{D}$.*

Theorem 3. *If $\lambda \geq 1$ and $\mu \geq 1+\sqrt{3}$, then $\mathbb{W}_{\lambda,\mu} \in \widetilde{\mathcal{S}}^*(\alpha)$, where*

$$\alpha = \frac{2}{\pi} arc\ sin \left(\eta\sqrt{1-\frac{\eta^2}{4}} + \frac{\eta}{2}\sqrt{1-\eta^2} \right),$$

for $\eta = 2(\mu+1)/\mu^2$.

Theorem 4. *For all $\lambda > -1$, $\mu > 0$, let $\mathbb{W}_{\lambda,\mu}(z)$ satisfy the inequality*

$$|z\mathbb{W}_{\lambda,\mu}(z)| < \frac{M(M-|a|)}{(M+1)(M+|a|)} \quad (0 \leq |a| < M \leq 1;\ z \in \mathbb{D}).$$

Suppose also ϕ be the (unique) solution of the initial value problem

$$\phi^{(n+1)}(z) \pm \mathbb{W}_{\lambda,\mu}(z)\phi^{(n)}(z) = \mathbb{W}_{\lambda,\mu}(z), \quad z \in \mathbb{D}$$

$(n \in \mathbb{N} \cup \{0\}$, $\phi(0) = 0$, $\phi'(0) = 1$, $\phi^{(k)}(0) = 0$ $(k = 2, \cdots, n-1)$, $\phi^{(n)}(0) = = a)$, where $\phi^{(n)}$ denotes n^{th} derivative with respect to z. Then the inequality $|\phi^{(n)}(z)| < M$ holds.

Theorem 5. *For each real $\lambda \geq 1$, $\mu \geq 1$ such that $\Gamma(\lambda+\mu) \geq 2\Gamma(\mu)$*

$$\text{Re}\left(\frac{\mathbb{W}_{\lambda,\mu}(z)}{z}\right) > \frac{1}{2}, \quad z \in \mathbb{D}.$$

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ОПРЕДЕЛЯЮЩИЕ МНОЖЕСТВА В ПРОСТРАНСТВЕ ГОЛОМОРФНЫХ В ШАРЕ ФУНКЦИЙ ПОЛИНОМИАЛЬНОГО РОСТА[1]

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Пусть $\mathbb{B}^N$ — единичный шар в $\mathbb{C}^N$. Полагаем $\mathbb{B}^1 =: \mathbb{D}$ — единичный круг в комплексной плоскости. Для каждого $p \geq 0$ образуем банахово пространство

$$A^{-p}(\mathbb{B}^N) := \{f \in H(\mathbb{B}^N) : \|f\|_p := \sup_{|z|<1} |f(z)|(1-|z|)^p < \infty\},$$

где $H(\mathbb{B}^N)$ — пространство всех голоморфных в $\mathbb{B}^N$ функций.

В соответствии с определением, введенным Ч. Горовицем, Б. Коренблюмом и Б. Пинчуком [1] при $N = 1$, подмножество S в $\mathbb{D}$ называется *определяющим* (*sampling*) для $A^{-\infty}(\mathbb{D})$, если

$$T_S(f) := \lim_{\zeta\in S, |\zeta|\to 1} \frac{\ln|f(\zeta)|}{|\ln(1-|\zeta|)|} = \lim_{|z|\to 1-0} \frac{\ln|f(z)|}{|\ln(1-|z|)|} =: T(f) \qquad (1)$$

для любой функции $f \in A^{-\infty}(\mathbb{D})$. Ясно, что это понятие без изменений распространяется на случай $N > 1$.

В работе [1] в основном изучался вопрос о взаимосвязи между определяющими множествами для $A^{-\infty}(\mathbb{D})$ и $A^{-p}(\mathbb{D})$. При этом, $S \subset \mathbb{D}$ называется *определяющим* (*sampling*) для пространства $A^{-p}(\mathbb{D})$, если имеется такая постоянная $C > 0$, что $\|f\|_p \leq C\|f\|_{p,S}$ для всех $f \in A^{-p}(\mathbb{D})$. Было показано, что если S является определяющим для всех $A^{-p}(\mathbb{D})$, $p > 0$, то оно будет определяющим и для $A^{-\infty}(\mathbb{D})$. Было также установлено,

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